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Exact renormalization group flow for ultracold Fermi gases in the unitary limit

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Abstract
We study the exact renormalization group flow for ultracold Fermi gases in the unitary regime. We introduce a pairing field to describe the formation of the Cooper pairs and take a simple ansatz for the effective action. A set of approximate flow equations for effective couplings including boson and fermionic fluctuations is derived. At some value of the running scale, the system undergoes a phase transition to a gapped phase. The values of the energy density, chemical potential, pairing gap and the corresponding proportionality constants relating the interacting and non-interacting Fermi gases are calculated. Standard mean-field results are recovered if we omit the boson loops.

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The physics of ultracold fermionic gases has recently drawn much attention as it provides an exciting possibility of studying the regime where the dynamics of the many-body system becomes independent of the microscopical details of the underlying interaction between two fermions. This regime can be probed using the technique of Feshbach resonances [1] when the scattering length is tuned to be much larger than the average inter-particle separation. The idealized case of the infinite scattering length a is often referred to as the unitary regime (UR). In the $a \to -\infty$ limit, the ground-state energy per particle is proportional to that of the non-interacting Fermi gas:

$$E_{GS} = \xi E_{FG} = \xi \frac{3}{5} \frac{k_F^2}{2M} = \xi \frac{3}{5} E_F, \tag{1}$$

where M and k_F are the fermion mass and Fermi momentum correspondingly, and ξ is the universal proportionality constant, which does not depend on the details of the interaction or type of fermions. The other dimensional characteristics of the cold Fermi gas in the UR such as paring energy Δ or chemical potential μ can also be represented in the same way:

$$\mu = \eta E_{FG}, \qquad \Delta = \epsilon E_{FG}. \tag{2}$$

The infinite scattering length implies non-perturbative treatment. The most 'direct'

non-perturbative method is based on the lattice field theory [2, 3]. However, being potentially the most powerful approach, lattice simulations still have many limitations related to finite-size effects, discretization errors, etc, which may even become amplified in certain physical situations (the system of several fermion species is one possible example). All that makes the development of the analytic approaches indispensable. Several such approaches have been suggested so far. The (incomplete) list includes the effective field theory (EFT) motivated formalism exploring the systematic expansion in terms of dimensionality of space [4] and a somewhat similar approach, based on the $1/N$ expansion [5]. More phenomenological approaches using the density functional method and many-body variational formalism have been developed in [6, 7].

In this paper, we consider the cold Fermi gas in the UR in the framework of the exact renormalization group (ERG) method suggested in [8] and applied to the non-relativistic many-fermion system with pairing in [9]. Different aspects of the ERG approach to non-relativistic systems have also been extensively studied in several subsequent papers [10–13]. Being spiritually related to the EFT-based approaches the ERG formalism is however fully non-perturbative and makes use of EFT as a guide to choose the ansatz for the effective action and to fix boundary conditions. The technical details of the approach in the context of the non-relativistic many-fermion systems were described in [10, 12] so that here we may give only a short account of the formalism.

The central object of the ERG formalism is the average effective action Γ_k which coincides with the bare action at the beginning of the evolution when the scale $k = \Lambda$ (with Λ being a starting scale) and is a full quantum action when $k = 0$. The average effective action satisfies the following flow equation,

$$\partial_k \Gamma = -\frac{\mathrm{i}}{2}\,\mathrm{Tr}[(\partial_k R)\,(\Gamma^{(2)} - R)^{-1}], \tag{3}$$

where

$$\Gamma^{(2)} = \frac{\delta^2 \Gamma}{\delta\phi_c \delta\phi_c} \tag{4}$$

and $R_k(q)$ is the IR regulator. We require $R_k(q)$ to vanish rapidly for $q^2 \gg k^2$ to ensure the results to be independent of the regulator at the vanishing scale. We also require $R_k(q)$ to behave as $R_k(q) \simeq k^2$ for $q^2 \ll k^2$ so that for large scale the regulator acts as an effective mass. To find a (approximate) solution of the ERG flow equations, one needs to choose the ansatz for the effective action and fix the corresponding boundary conditions represented by the bare action. There are no strict quantitative criteria for choosing the ansatz so that it seems reasonable to work with the 'relevant' degrees of freedom and include the interaction terms satisfying (and allowed by) all possible symmetry constraints. In our case, the degrees of freedom are the strongly interacting fermions in the limit of the infinite scattering length. At the starting scale, the medium parameters such as Fermi momenta, chemical potentials, etc, play a little role so that at this scale the average effective action is just the bare action with the standard four-fermion interaction in vacuum. One possible choice for the bare action is the simplest attractive EFT-motivated four-fermion pointlike interaction with the Lagrangian

$$\mathcal{L}_i = -\tfrac{1}{4}\, C_0 \big(\psi^\dagger \sigma_2 \psi^{\dagger \mathrm{T}}\big)\big(\psi^{\mathrm{T}} \sigma_2 \psi\big). \tag{5}$$

With a decreasing scale the role of the many-body effects becomes more and more important and at some scale one may anticipate the Cooper instabilities, symmetry breaking, formation of the correlated fermion pairs, etc, to occur. Taking all that into account we write

the ansatz for the effective action Γ_k in the following form:

$$\Gamma[\psi,\psi^\dagger,\phi,\phi^\dagger]=\int d^4x\,\left[\phi^\dagger(x)\left(Z_\phi\,\mathrm{i}\partial_t+\frac{Z_m}{2m}\,\nabla^2\right)\phi(x)-U(\phi,\phi^\dagger)\right.$$
$$+\psi^\dagger\left(Z_\psi(\mathrm{i}\partial_t+\mu)+\frac{Z_M}{2M}\,\nabla^2\right)\psi$$
$$\left.-g\left(\frac{\mathrm{i}}{2}\,\psi^{\mathrm{T}}\sigma_2\psi\phi^\dagger-\frac{\mathrm{i}}{2}\,\psi^\dagger\sigma_2\psi^{\dagger\mathrm{T}}\phi\right)\right]. \tag{6}$$

We introduce the effective pairing field ϕ and use the standard Hubbard–Stratonovich transformation to cancel the four-fermion interaction. The couplings $Z_{(m,M,\phi,\psi)}$ and g should in principle run with the scale. We also include the kinetic term for the pairing field needed to compute the boson loop contributions. Note that our definition of the effective potential includes the term $-2\mu Z_\phi\phi^\dagger\phi$ which describes the coupling of the pairing field to the chemical potential. We expand the effective potential about its minimum

$$U(\phi,\phi^\dagger)=u_0+u_1(\phi^\dagger\phi-\rho_0)+\tfrac{1}{2}\,u_2(\phi^\dagger\phi-\rho_0)^2, \tag{7}$$

where u_n are defined at the minimum of the potential, $\phi^\dagger\phi=\rho_0$. The coefficients at the quadratic (in fields) term determine the phase of the system. When $u_1>0$, the system is in the symmetric phase with a trivial vacuum $\rho_0=0$. At some critical scale, the coefficient u_1 approaches zero and the system undergoes the transition to the broken (or condensed) phase with $\rho_0\neq 0$ so that in this phase

$$U(\phi,\phi^\dagger)=u_0+\tfrac{1}{2}\,u_2(\phi^\dagger\phi-\rho_0)^2+\cdots. \tag{8}$$

The renormalization factors can also be expanded about $\rho=\rho_0$:

$$Z_\phi(\phi,\phi^\dagger)=z_{\phi 0}+z_{\phi 1}(\phi^\dagger\phi-\rho_0)+\cdots. \tag{9}$$

The other renormalization factors can be expanded in the same way.

In this paper, we take into account the terms in the expansion of the effective potential up to quartic order in the fields. The minimum ρ_0 of the effective potential evolves with the scale in the condensed phase and all the coefficients of the expansion should depend on both the scale and ρ_0. There are few options of how to organize the evolution of the system. The choice would depend on the physical quantities which one would like to get at the end of the evolution. The obvious and the most general one is just to run all quantities of interest. The other (simpler) way to extract essentially the same information is to fix some parameters, such as the chemical potential or particle number density, and run the rest. Fixed particle number density and evolving chemical potential seems more interesting as it gives the potential possibility of going to the BEC regime where the chemical potential eventually becomes negative. In this case, the coefficients u_n and the renormalization factors will depend on the running scale both explicitly and implicitly via the dependencies on $\rho_0(k)$ and $\mu(k)$. We can, therefore, define a total derivative

$$\frac{d}{\mathrm{d}k}=\partial_k+\frac{\mathrm{d}\rho_0}{\mathrm{d}k}\,\frac{\partial}{\partial\rho_0}+\frac{\mathrm{d}\mu}{\mathrm{d}k}\,\frac{\partial}{\partial\mu}. \tag{10}$$

Applying this to $n=-\partial U/\partial\mu$ at $\rho=\rho_0$ and assuming the constant particle number density we get

$$-2z_{\phi 0}\,\frac{\mathrm{d}\rho_0}{\mathrm{d}k}+\chi\,\frac{\mathrm{d}\mu}{\mathrm{d}k}=-\left.\frac{\partial}{\partial\mu}(\partial_k U)\right|_{\rho=\rho_0}, \tag{11}$$

where we defined

$$\chi=\left.\frac{\partial^2 U}{\partial\mu^2}\right|_{\rho=\rho_0}. \tag{12}$$

The other ERG flow equations can be obtained in a similar way so that after some algebra we get

$$\frac{\mathrm{d}u_0}{\mathrm{d}k} + n\,\frac{\mathrm{d}\mu}{\mathrm{d}k} = \partial_k U|_{\rho=\rho_0}\,, \tag{13}$$

$$-u_2\,\frac{\mathrm{d}\rho_0}{\mathrm{d}k} + 2z_{\phi 0}\,\frac{\mathrm{d}\mu}{\mathrm{d}k} = \frac{\partial}{\partial\rho}(\partial_k U)\bigg|_{\rho=\rho_0}\,, \tag{14}$$

$$\frac{\mathrm{d}u_2}{\mathrm{d}k} - u_3\,\frac{\mathrm{d}\rho_0}{\mathrm{d}k} + 2z_{\phi 1}\,\frac{\mathrm{d}\mu}{\mathrm{d}k} = \frac{\partial^2}{\partial\rho^2}(\partial_k U)\bigg|_{\rho=\rho_0}\,, \tag{15}$$

$$\frac{\mathrm{d}z_{\phi 0}}{\mathrm{d}k} - z_{\phi 1}\,\frac{\mathrm{d}\rho_0}{\mathrm{d}k} + \frac{1}{2}\,\chi'\,\frac{\mathrm{d}\mu}{\mathrm{d}k} = -\frac{1}{2}\,\frac{\partial^2}{\partial\mu\partial\rho}(\partial_k U)\bigg|_{\rho=\rho_0}\,, \tag{16}$$

where we have defined

$$\chi' = \frac{\partial^3 U}{\partial\mu^2\partial\rho}\bigg|_{\rho=\rho_0}\,. \tag{17}$$

Note that we introduce the coefficient u_3 which corresponds to the higher order terms in the expansion of the effective potential. This coefficient occurs when we act on $\frac{\partial^2}{\partial\rho^2}U$ with the above-defined total derivative.

The set of evolution equations in the symmetric phase can easily be recovered using the fact that the chemical potential does not run in the symmetric phase and that $\rho_0 = 0$. All the higher order terms such as χ, χ', u_3 and $z_{\phi 1}$ were calculated from the mean-field-type expression when the boson loops are neglected and the effective potential can be calculated explicitly (see below). The functions $\rho_0(k)$ and $\mu(k)$ which determine the physical energy gap and chemical potential in the limit $k \to 0$ were computed in [12]. In this paper, we focus on the field-independent part of the effective potential u_0 which is related to the energy density of the Fermi gas in the UR. As can be seen from the evolution equations, the coefficient u_0 is not coupled to the rest but its value depends on running chemical potential $\mu(k)$ so that to find u_0 we have to solve the whole system of the flow equations.

The explicit expressions for the driving term $\partial_k U$ can be obtained from the effective action after the straightforward algebra and has the following form:

$$\partial_k U = -\int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\,\frac{E_{FR}}{\sqrt{E_{FR}^2+\Delta^2}}\,\partial_k R_F + \frac{1}{2Z_\phi}\int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\,\frac{E_{BR}}{\sqrt{E_{BR}^2 - V_B^2}}\,\partial_k R_B, \tag{18}$$

where

$$E_{BR}(q,k) = \frac{Z_m}{2m}\,q^2 + u_1 + u_2(2\phi^\dagger\phi - \rho_0) + R_B(q,k), \qquad V_B = u_2\phi^\dagger\phi = u_2\rho \tag{19}$$

and

$$E_{FR}(q,k,\mu) = \frac{Z_M}{2M}\left(q^2 - p_F^2\right) + R_F(q,k,\mu). \tag{20}$$

The other driving terms can be obtained by taking the corresponding derivatives of $\partial_k U$ with respect to boson fields. The expressions for u_1 and u_2 can be obtained in a rather trivial way but the derivation of the driving term for the wavefunction renormalization factor Z_ϕ is worth discussing. To calculate Z_ϕ, we need to consider a time-dependent background field. Taking

$$\phi(x) = \phi_0 + \eta\,\mathrm{e}^{-\mathrm{i}p_0 t}, \tag{21}$$

where η is a constant, we can get the evolution of Z_ϕ from

$$\partial_k Z_\phi = \frac{1}{\mathcal{V}_4}\,\frac{\partial}{\partial p_0}\left(\frac{\partial^2}{\partial\eta\partial\eta^\dagger}\,\partial_k\Gamma\right)_{\eta=0}\Bigg|_{p_0=0}. \tag{22}$$

Defining

$$\mathbf{\Gamma}^{(3)}_{BB\phi} = \frac{\partial}{\partial\phi}\,\mathbf{\Gamma}^{(2)}_{BB} = \begin{pmatrix} -2u_2\phi^\dagger & -2u_2\phi \\ 0 & -2u_2\phi^\dagger \end{pmatrix} \tag{23}$$

and

$$\mathbf{\Gamma}^{(3)}_{FF\phi} = \frac{\partial}{\partial\phi}\,\mathbf{\Gamma}^{(2)}_{FF} = \begin{pmatrix} 0 & \mathrm{i}g\sigma_2 \\ 0 & 0 \end{pmatrix}, \tag{24}$$

one can write the corresponding part of the evolution equation in the form

$$\begin{aligned}\frac{\partial^2}{\partial\eta\partial\eta^\dagger}\,\partial_k\Gamma\Big|_{\eta=0} &= +\mathrm{i}\,\mathrm{Tr}\left[(\partial_k\mathbf{R}_F)\left(\mathbf{\Gamma}^{(2)}_{FF}-\mathbf{R}_F\right)^{-1}\mathbf{\Gamma}^{(3)\dagger}_{FF\phi}\left(\mathbf{\Gamma}^{(2)}_{FF}-\mathbf{R}_F\right)^{-1}\mathbf{\Gamma}^{(3)}_{FF\phi}\left(\mathbf{\Gamma}^{(2)}_{FF}-\mathbf{R}_F\right)^{-1}\right] \\ &\quad -\mathrm{i}\,\mathrm{Tr}\left[(\partial_k\mathbf{R}_B)\left(\mathbf{\Gamma}^{(2)}_{BB}-\mathbf{R}_B\right)^{-1}\mathbf{\Gamma}^{(3)\dagger}_{BB\phi}\left(\mathbf{\Gamma}^{(2)}_{BB}-\mathbf{R}_B\right)^{-1}\mathbf{\Gamma}^{(3)}_{BB\phi}\left(\mathbf{\Gamma}^{(2)}_{BB}-\mathbf{R}_B\right)^{-1}\right].\end{aligned} \tag{25}$$

Note that $\mathbf{R}_{F(B)}$ is the diagonal matrix consisting of fermion (boson) regulators. After the explicit calculations of the above traces we obtain the following.

Putting it all together we get

$$\begin{aligned}\frac{1}{\mathcal{V}_4}\,\frac{\partial}{\partial p_0}\left(\frac{\partial^2}{\partial\eta\partial\eta^\dagger}\,\partial_k\Gamma\right)_{\eta=0}\Bigg|_{p_0=0} &= -\frac{g^2}{4}\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\,\frac{2E^2_{FR}-\Delta^{(c)2}}{\left(E^2_{FR}+\Delta^{(c)2}\right)^{5/2}}\,\mathrm{sgn}\,(q-p_\mu)\,\partial_k R_F \\ &\quad -\frac{u_2V^{(c)}_B}{2}\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\,\frac{2E^{(c)2}_{BR}-6E^{(c)}_{BR}V^{(c)}_B+V^{(c)2}_B}{\left(E^{(c)2}_{BR}-V^{(c)2}_B\right)^{5/2}}\,\partial_k R_B.\end{aligned} \tag{26}$$

As a check on this result, we note that u_1 contains a piece $-2\mu Z_\phi$. Hence we can also obtain the evolution of Z_ϕ from

$$-\frac{1}{2}\,\frac{\partial^2}{\partial\mu\partial\rho}(\partial_k\bar{U})\Big|_{\rho=\rho_0}. \tag{27}$$

Taking the partial derivative with respect to μ one can see that the result we obtain from equation (26) agrees with equation (27).

The energy density extracted from equation (18) for the effective potential contains the field-independent contribution which is proportional to Λ so that the evolution equation as is written above is still not enough to extract the physical energy density. Therefore, one needs to make a subtraction based on a physical assumption that the energy density must be a constant, equal to that of the free Fermi gas when $u_1 = u_2 = \cdots = u_n = 0$. The modified flow equation for the effective potential can be written as

$$\partial_k U = -\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\left(1-\frac{E_{FR}}{\sqrt{E^2_{FR}+\Delta^2}}\right)\partial_k R_F + \frac{1}{2Z_\phi}\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\left(1-\frac{E_{BR}}{\sqrt{E^2_{BR}-V^2_B}}\right)\partial_k R_B. \tag{28}$$

Note that the flow does not change as the added term is field independent. We utilize the type of the cut-off functions suggested first in [14](see further discussion in [15]) for the boson ERG flow and in [10] for the fermionic case. This form of the cut-off functions allows for the significant practical simplifications when calculating the loop diagrams. One notes that the

cut-off function for fermions can be written in different forms. However, all of them should contain the sgn function reflecting the particle and/or hole 'faces' of the in-medium fermion. In this paper, we use the cut-off function in the form, considered in [13],

$$R_F = \frac{1}{2M}\left[\left(k^2 sgn(q-p_\mu) - \left(q^2 - p_\mu^2\right)\right)\right]\theta\left(k^2 - \left|q^2 - p_\mu^2\right|\right), \tag{29}$$

where $p_\mu = (2M\mu)^{1/2}$, and

$$R_B = \frac{1}{2m}(k^2 - q^2)\theta(k-q). \tag{30}$$

The boson loops can be neglected at high scale, and the flow equation can be integrated explicitly resulting in the following expression for the effective potential in the mean-field (MF) approximation,

$$U^{\mathrm{MF}}(\rho,\mu,k) = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}(E_{FR}(q,\mu,k) - \sqrt{E_{FR}(q,\mu,k)^2 + g^2\rho}) + C, \tag{31}$$

where C is the constant of integration. After some algebra the expression for U^{MF} can be rewritten as

$$U^{\mathrm{MF}}(\rho,\mu,k) = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\left(E_{FR}(q,\mu,k) + \frac{g^2\rho}{2\epsilon_q} - \sqrt{E_{FR}(q,\mu,k)^2 + g^2\rho}\right) - \frac{Mg^2\rho}{4\pi a}, \tag{32}$$

where $\epsilon_q = E_{FR}(q,0,0)$.

We note again that all the higher order couplings u_3, χ, χ' have been calculated from $U^{\mathrm{MF}}(\rho,\mu,k)$ by taking the appropriate derivatives.

Differentiating with respect to ρ and setting the derivative and running scale equal to zero, we find that Δ^2 at the minimum satisfies

$$-\frac{M}{4\pi a} + \frac{1}{2}\int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\left[\frac{1}{E_{FR}(q,0,0)} - \frac{1}{\sqrt{E_{FR}(q,p_F,0)^2 + \Delta^2}}\right] = 0. \tag{33}$$

This is exactly the gap equation derived, for example, in [16].

To get the number density of fermions, we can differentiate $U(\rho,\mu,0)$ with respect to μ. This gives

$$n = \int \frac{d^3\boldsymbol{q}}{(2\pi)^3}\left[1 - \frac{E_{FR}(q,p_F,0)}{\sqrt{E_{FR}(q,p_F,0)^2 + \Delta^2}}\right], \tag{34}$$

again in agreement with [16]. One can therefore conclude that the standard MF result can be reproduced within ERG if the boson fluctuations are neglected.

The boundary conditions for the coefficients u_i can be obtained by differentiating the expression for U^{MF} with respect to ρ at the starting scale $k = \Lambda$.

It is worth mentioning that in this paper we include running of $\mu, \rho_0, u_i's$ and the renormalization factor Z_ϕ. This is the minimum set of running parameters needed to go beyond the mean-field approximation. The other couplings are held fixed at their initial values.

The useful quantity to check the consistency of the approach is the boson scattering length a_B. It is well known that the MF calculations lead to the relation $a_B = 2a_F$, where a_F is the fermion scattering length. Deviation from this result is due to the boson loop effects and therefore goes beyond the MF approximation. In our approach, the boson scattering length is given by the relation $a_B = 2u_2/Z_\phi^2$. Using the cut-off in the form specified in equation (22)

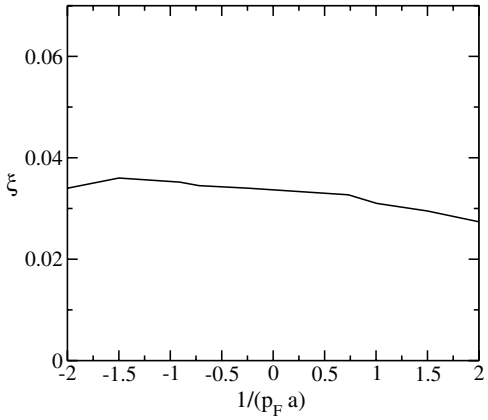

Figure 1. Deviation of the chemical potential from its MF value as a function of $1/(p_F a)$.

and calculating the values of u_2 and Z_ϕ^2 in the MF approximation, it can easily be demonstrated that the relation $a_B = 2a_F$ is indeed satisfied.

The calculations with boson loops lead to the relation $a_B = 1.13a_F$. It is still quite far away from the relation $a_B = 0.6a_F$ found in the full four-body calculations of [17]. It means that the present truncation, while providing useful tool to go beyond the MF level, is still too crude to realistically describe the effects of boson loops at least for the boson scattering length. We note in passing that neither the Yukawa coupling nor fermionic renormalization constants run in vacuum. It is worth mentioning that similar ERG studies [13] resulted in relation $a_B = 0.91a_F$. The nature of the difference between two otherwise similar calculations is not clear at present. This issue clearly requires further investigations which involves a detailed comparison of two approaches.

The boson loop contributions are found to be small in the unitary regime when medium effects are included. It looks somewhat puzzling if we compare this result with those obtained for the boson–boson scattering length. We note however that universal constants seem to be less sensitive to the boson loops effect than boson–boson scattering amplitude. The possible physical reason is that the boson rescattering effects will be partially suppressed in medium. A more definite conclusion can be drawn after a number of other effects, for example the evolution of all the couplings, have been taken into account.

Note that it is important to keep boson loops for theoretical consistency as it leads to the convex effective potential. The point is that in the vicinity of the physical point $k = 0$ the coupling u_2 turns out to be vanishingly small so that the potential becomes flat with the minimum shifted from the origin. We emphasize that the effective potential retains its 'mexican hat' form at any finite scale. With a decreasing scale the bump becomes less and less pronounced so the effective potential eventually evolves into the convex form. Without the boson loop contribution the u_2 coupling is finite at $k \to 0$ and not small so that the convexity property of the effective potential is missing in agreement with the known results.

In addition, smallness of the boson loop contributions in the unitary regime qualitatively agrees with the conclusion made in [18], where the effects of boson loops were included via the corresponding self-energy corrections. According to [18], the boson loops contribution at zero temperature is relatively small both in the unitary regime and in the BCS/BEC regions. We show in figure 1 the deviation of the full chemical potential from its MF value as a function

Table 1. Universal coefficients.

References	ξ	ϵ	η
[2]	0.42	0.9	0.71
[24]	0.22		
[3]	0.41		0.7
[25]	0.44	0.93	
[26]		1.03	
[4]	0.39		
[27]	0.3	0.66	
[30]	0.55		

of $(p_F a)^{-1}$ in the BEC regime. This deviation is defined as $\xi = (\mu(\mathrm{MF}) - \mu(\mathrm{Full}))/E_F$. It is seen from figure 1 that this quantity is small compared to the typical value of the chemical potential ($\mu \sim O(1)$ at $(p_F a)^{-1} \simeq 1$). We emphasize that, although we believe that this conclusion is qualitatively correct, the actual value of the 'beyond-mean-field' corrections can be different if running of all the couplings is included. Besides, including higher order terms in the effective action can further change the contributions from the 'beyond-mean-field' effects. All that constitutes the subject for the future studies. We stress that, despite being clearly subleading order, the boson loops contribution to the physical observables calculated in [18] is still larger than that obtained in this paper. To understand the cause of this deviation, one needs to establish one-to-one diagrammatic correspondence between ERG and the more traditional approach adopted in [18]. It is a difficult and still unsolved problem as ERG includes, in principle, an infinite (and probably mixed) set of diagrams so that one-to-one correspondence between different contributions in the ERG approach and certain classes of diagrams may simply not exist.

Now we turn to the results. First observation is that the results become practically independent on the starting scale Λ if $\Lambda \geqslant 10 p_F$. The phase transition to the condensed phase occurs at $k_{\mathrm{crit}} \simeq p_F$. One notes that the calculations with other types of cut-offs, both sharp and smooth, lead to relatively close values of k_{crit} [9, 12]. The fact that all values of k_{crit} are clustered around the value of k_F makes good sense as at this scale the system becomes sensible to the medium effects such as Cooper instabilities, gap formation, etc.

We found the values of 0.62, 0.96 and 1.11 for the universal coefficients ξ, η and ϵ correspondingly. The calculations without boson loops give $\xi_{MF}(\eta_{MF}, \epsilon_{MF}) = 0.65(0.98, 1.14)$. One notes that these values satisfy the relation $\Delta \sim 2E_{GS}$ found in [2]. The effect of the boson loops is found to be small for both the optimized cut-off function used in this paper and the smoothed theta function used in [9]. One notes, however, that two cut-offs lead to different signs of the 'beyond-mean-field' contribution. In the ideal case all the cut-offs should, of course, lead to the same results but in practice the unavoidable truncation of the effective action will always lead to some uncertainties. Taking into account the smallness of the boson loop effects it is hard to see if this sign uncertainty is the result of truncation or just numerical instabilities which are known to be larger for expressions involving step functions.

The obtained value for ξ is close to the experimental data from [19], $\xi = 0.74(7)$. The other measurements give $\xi = 0.34(15)$ and $\eta = 0.6(15)$ [20]; $\xi = 0.32^{+0.13}_{-0.10}$ and $\eta = 0.53^{+0.13}_{-0.10}$ [21]; $\xi = 0.46(5)$ and $\eta = 0.77(5)$ [22]; $\xi = 0.51(4)$ and $\eta = 0.85(4)$ [23]. The results of theoretical calculations [4, 2, 24–27, 3, 13] are summarized in table 1.

As one can see both experiment and numerical simulations do not provide the coherent value of the ξ constant so it is difficult to judge the quality of the numerical estimates provided

by the ERG calculations. One may only conclude that the ERG approach leads to the sensible values of the universal coefficients consistent with the experiment and lattice calculation but a more detailed comparison can be done when more accurate date are obtained. We note, however, that the values of the universal parameters are still somewhat higher than the 'world average'. One possible cause could be the neglect of the screening effects [28] which are known to decrease the values of the gap and energy density. Naive extrapolation of our results using the known value of the Gorkov–Melik–Barkhudarov's correction [28] indeed brings the values of the universal coefficients closer to the 'world average' of the lattice and experimental data. Clearly, this point requires further analysis and the corresponding work is now in progress [29] (see also [30]). There are several other directions where the current ERG approach can further be developed. First, as mentioned above, running of all coupling constants should be included. Second, the calculations with the entire effective potential taking into account the screening effects should also be performed [31]. Among the other physical applications one could mention the extension to the finite-temperature case and analysis of the deviations from the unitary limit.

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